Probability Theory

Monday, January 23, 2023 3:10 AM

Continuous Random Variables

A random variable X can take an uncountable range of values.

The Expect value (Mean, Average, Centroid) can be calculated with:

Discrete random variables:

$E[X] = \sum xp(x)dx$

Continuous random variables:

$$E[X] = \int_{-\infty}^{\infty} xp(x)dx$$

The Variance is the spread around the mean, the distance from X to the average value

$Var(X) = E[X^2] - E[X]^2$

The Standard Deviation represents the distance between each unit.

$Std = \sqrt{Var(x)}$

The Covariance measures the relationship between two random variables

$CoV(X,Y) = E\big[(X - E[X])(Y - E[Y])\big] = E[XY] - E[X]E[Y]$

Uniform Distribution:

This distribution gives the same probability p to all values of a random variable X, between a range [a,b] . Example a dice theorically has the same probability in all its posibble values. Then

$$P(X = x) = \begin{cases} p; x \in [a,b] \\ 0; x \notin [a,b] \end{cases}$$

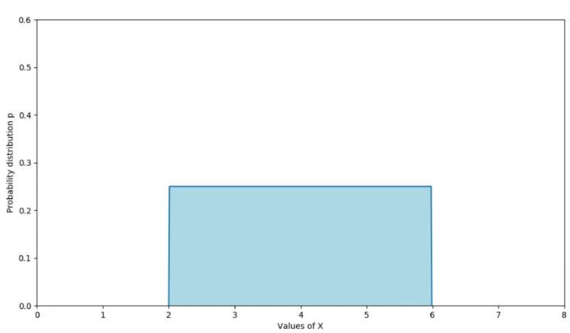
Equation:

$$p(x) = \frac{1}{b-a}$$

The probability distribution integral should be normalized to one.

$$\int_a^b p(x)dx = 1$$

$$\int_2^6 p(x)dx = \int_2^6 \frac{1}{6-2}dx = 0.25x\Big|_2^6 = 1$$



Cumulative Distribution:

This distribution gives more probability to the highest values of X.

A random value between a and b has to main that the sum of all the probabilities is 1.

$$\int_a^b p(x)dx = 1$$

We could use a triangle as a cumulative distribution:

$p(x)dx = m(x - a)$

Where a is the intersection with the x axis, and m is the slope of the line

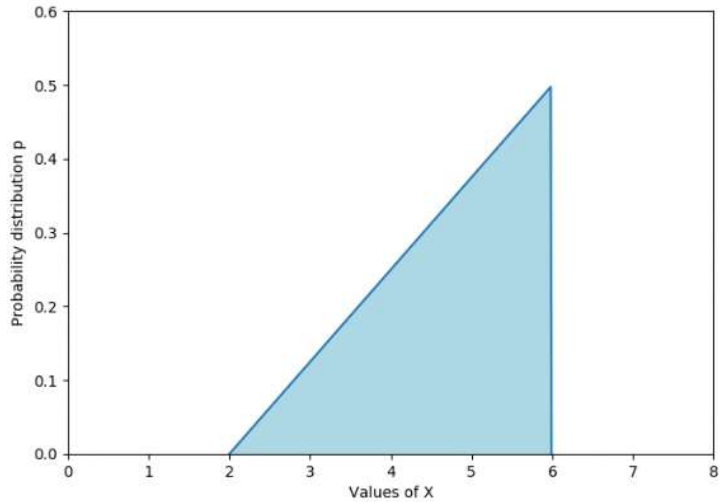
$$\int_a^b p(x)dx = \int_a^b m(x-a)dx = 1$$

$$\left.\frac{m(x-a)^2}{2}\right|_a^b = \frac{m(b-a)^2}{2} = 1$$

$$m = \frac{2}{(b-a)^2}$$

Example for a distribution with a=2 and b=6:

$$m = \frac{2}{16} = 0.125$$



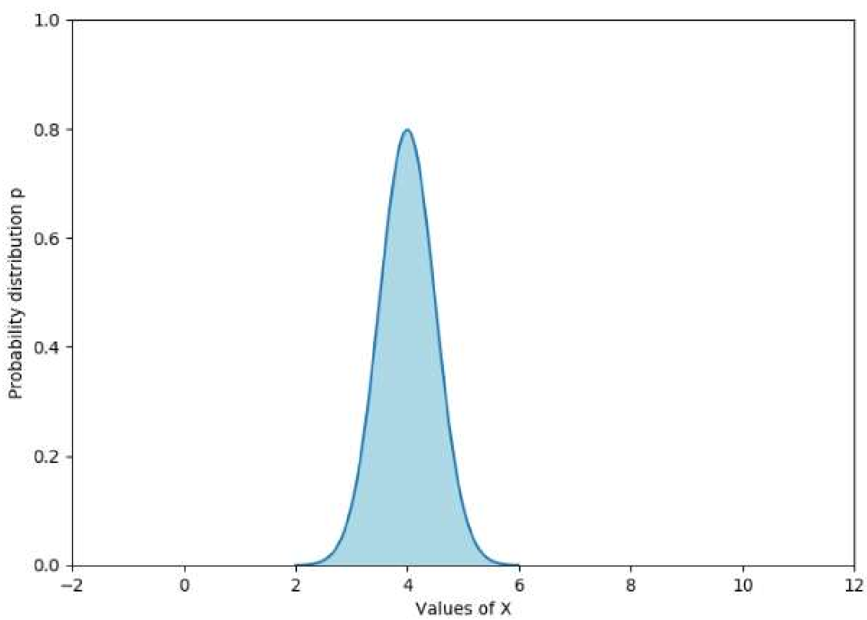
Gaussian Distribution:

It is a continuous distribution with a high peak in the middle. It can be denoted as:

$X \sim N(\mu, \sigma)$

The PDF (Probability Density Function) is defined as:

$$p(x;\mu;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



State of a Robot

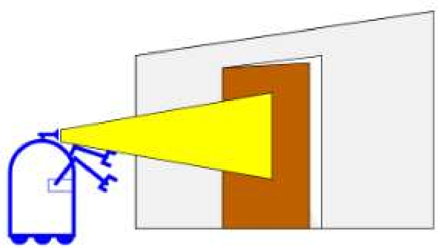
The general state of a robot can be expressed as:

$p(x_t | x_{0:t-1}, z_{1:t-1}, u_{1:t})$
Where:
x_t: *State of robot at time t*
$x_{0:t-1}$: *Previous robot states*
$z_{1:t-1}$: *Previous robot measurements*
$u_{1:t}$: *Previous and actual actions of robot*
A Markov Model is used to represent the system, so the future robot states won't depend on the old previous states since the beginning. Then the new equation will be:
$p(x_t | x_{t-1}, z_{t-1}, u_t)$

Belief distribution:
The internal state of a robot can be modeled as a belief distribution:
$bel(x_t) = p(x_t, z_{1:t}, u_{1:t})$
This will take into account the previous measurements and actions. Measurement will decrease the uncertainty, and action will increase it.

Example of the effect of a measurement z

Imagine a robot in front of a door.



Given a measure z obtained by the robot (z could be the data from the laser for example), what is the belief that the door is $open$?

- We are looking for $P(open | z = senses\ open)$
- The door has two possible states: $\{open\ ,\ closed\}$ that are equally possible:
  - $P(open) = P(closed) = 0.5$
- The sensor of z gives the probabilities:
  - $P(z = senses\ open | open) = 0.6$, $P(z = senses\ closed | open) = 0.4$
  - $P(z = senses\ open | closed) = 0.3$, $P(z = senses\ closed | closed) = 0.7$
- Evidence:
  $P(z) = P(z|open) \cdot P(open) + P(z|closed) \cdot P(closed)$

Finally, with Bayes' rule:

$$P(open | z = senses\ open) = \frac{P(z = senses\ open | open) \cdot P(open)}{P(z = senses\ open)} = \frac{P(z = senses\ open | open) \cdot P(open)}{P(z = senses\ open | open) \cdot P(open) + P(z = senses\ open | closed) \cdot P(closed)} = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{0.6}{0.9} = 0.67$$

The robot has a 67% belief that the door is open if the measurement z was that it sensed the door was open.